(26) and (27), by introducing additional hypothetical sources to account for heat reflected from the corresponding boundary planes. In most cases, the IS semiconductor crystal has dimensions $(0.5 \times 0.5 + 4 \times 4) \cdot 10^{-6}$ m^2 and, therefore, for the calculation of temperature fields during pulse microwelding, calculational schemes with one and three hypothetical sources are adequate.

NOTATION

T, temperature; r_0, radius of the heat source; h, crystal thickness; λ, thermal conductivity; x, y, z, Cartesian coordinates; q_0, specific heat flux; a, b, semiaxes of the ellipse; dS, elementary area of the ellipse; r, radial variable; α, heat-transfer coefficient; θ, dimensionless temperature; ξ, χ, dimensionless coordinates; Bi, Biot number; ε, emissivity; σ, Stefan—Boltzmann constant; T_0, initial temperature of the medium; k, l, distances from the center of the heat source to the crystal boundary planes.

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SIMULATION OF THE THERMAL STATE OF A BILLET FROM CASTING TO ROLLING

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A mathematical model is given for the thermal treatment of a billet for rolling, in which the crystallization, cooling, and reheating are considered as stages in a single process.

Very complex effects occur in the heating and cooling of a billet during preparation for rolling; the time during which such a billet is suitable for rolling is governed by the size, type of mold, grade of steel, time spent in the mold, cooling time after removal from the mold, temperature in the heating oven, and so on. Many of these parameters may vary within wide limits. The product quality and the throughput are dependent on the parameter values.

Methods are available for calculating the individual stages in the process, and one can use these to calculate the crystallization, cooling, and heating with reasonable precision, but only if one has available reasonably reliable data on the initial state for each of the stages.

However, it has been shown [1-4] that division of the preparation into stages cannot provide all the necessary information, and it would be best to perform the calculations on

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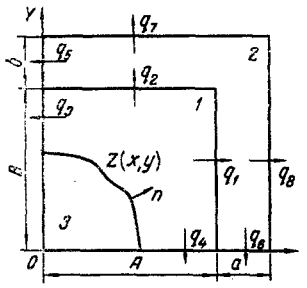

Fig. 1. Mathematical formulation: 1) solid; 2) mold; 3) melt.

the crystallization, cooling, and reheating in terms of a single process. Very few papers deal with the process from this viewpoint, and in view of the complexity they are usually based on various gross simplifications, which often substantially reduce the resulting accuracy.

One study [2] deals with the most economical modes of billet heating for hot rolling on the assumption that the movement of the heated front is in accordance with $Z = K_i\sqrt{\tau}$, where K_i is a constant dependent on the conditions of crystallization, cooling in the mold, subsequent cooling in air, movement through the heating oven, and so on.

A major disadvantage of this model is that any change in the conditions, for instance, the instant of removal from the mold, will immediately alter the law followed by the crystallization front. This neglects the fact that thermal processes show considerable lag, and the new speed of the front is then not determined by the current conditions, but has to be specified in advance. Further, the available evidence indicates that the speed of the front in general does not obey the square-root law.

Electrothermal analogs have also been used [4], but it is clear from another study [7] that this treatment is justified only when fairly low accuracy is acceptable and high working speed is not necessary.

A study has been made [5] of the effects of the initial billet temperature on heating time; the data on the thermal state of the billet during crystallization were derived from experiment, while the heating was calculated, which gave a reasonably full analysis for a completely solidified casting, but one which was very difficult to apply to the heating of a billet containing a liquid core. Also, complex experiments are required in order to apply the method at all.

Here we use a fuller mathematical model that allows one to consider various styles of billet handling in an actual oven, including reheating a billet containing a liquid core, which is particularly important in some high-speed processes.

The following thermal-conduction equation applies to the solidified part of the billet (Fig. 1):

$$c_1(t)\,\rho(t)\,\frac{\partial t_1(x, y, \tau)}{\partial \tau} = \operatorname{div}[\lambda(t)\operatorname{grad} t_1(x, y, \tau)] \tag{1}$$

subject to the initial condition

$$t_1(x, y, 0) = f_1(x, y). \tag{2}$$

We assume that the temperature of the liquid phase is constant at T_{cr}; any superheating in the liquid can be incorporated by increasing the latent heat of crystallization appropriately. Then the condition at the mobile boundary takes the form

$$\lambda_1(t)\frac{\partial t_1(z, \tau)}{\partial \bar{n}} = \rho L\frac{\partial z(x, y, \tau)}{\partial \tau}, \tag{3}$$

where $z(x, y, \tau)$ is the position of the phase interface and $\bar{n}$ is the vector for the normal to z, with

$$z(x, y, 0) = z_0(x, y). \tag{4}$$

There are no heat fluxes through the symmetry plane, i.e.,

$$q_3 = \lambda_1(t)\frac{\partial t_1(0, y, \tau)}{\partial x} = 0, \quad q_4 = \lambda_1(t)\frac{\partial t_1(x, 0, \tau)}{\partial y} = 0. \tag{5}$$

We assume also that the air-filled gap between the billet and the mold forms instantaneously after a certain time lapse and then remains constant. In that case, the heat transfer from the billet to the mold occurs by direct contact before the gap is formed, and by radiation and convection afterwards, with the inner surface of the mold acting as the external medium. When the mold is removed, and when the billet is in the reheating oven, the mode of heat transfer remains as before, the only changes being in the heat-transfer coefficient and in the parameters of the environment. Therefore, the heat-transfer conditions at the surface of the billet may be put as

$$q_1(\tau)=\lambda_1(t)\frac{\partial t_1(A, y, \tau)}{\partial x}=\begin{cases}\lambda_2(t)\dfrac{\partial t_2(A, y, \tau)}{\partial x}, & 0<\tau\leqslant\tau^*,\\ C[t_1(A, y, \tau)-T]+D[t_1^4(A, y, \tau)-T^4], & \end{cases} \tag{6}$$

$$q_2(\tau)=\lambda_1(t)\frac{\partial t_1(x, B, \tau)}{\partial y}=\begin{cases}\lambda_2(t)\dfrac{\partial t_2(x, B, \tau)}{\partial y}, & 0<\tau\leqslant\tau^*,\\ C[t_1(x, B, \tau)-T]+D[t_1^4(x, B, \tau)-T^4], & \end{cases} \tag{7}$$

where C, D, and T, respectively, take the values α_3, σ_3, t_s for $\tau^* \leq \tau \leq \tau_c$, α_1, σ_1, t_a for $\tau_c \leq \tau \leq \tau_{in}$ and α_2, σ_2, t_b for $\tau > \tau_{in}$

We solve the thermal-conduction equation to determine the temperature distribution in the mold:

$$c_2(t)\,\rho_2(t)\frac{\partial t_2(x\ y, \tau)}{\partial\tau}=\operatorname{div}[\lambda_2(t)\operatorname{grad}t_2(x, y, \tau)] \tag{8}$$

subject to the initial condition

$$t_2(x, y, 0)=f_2(x, y), \tag{9}$$

and the symmetry condition

$$q_5(\tau)=\lambda_2(t)\frac{\partial t_2(0, y, \tau)}{\partial x}=0,\quad q_6(\tau)=\lambda_2(t)\frac{\partial t_2(x, 0, \tau)}{\partial y}=0 \tag{10}$$

in conjunction with boundary conditions of radiation–convection type at the outer surface of the mold:

$$q_8(\tau)=\lambda_2(t)\frac{\partial t_2(A+a, y, \tau)}{\partial x}=\alpha_2[t_2(A+a, y, \tau)-t_c]+\sigma_2[t_2^4(A+a, y, \tau)-t_c^4], \tag{11}$$

$$q_7(\tau)=\lambda_2(t)\frac{\partial t_2(x, B+b, \tau)}{\partial y}=\alpha_2[t_2(x, B+b, \tau)-t_c]+\sigma_2[t_2^4(x, B+b, \tau)-t_c^4]. \tag{12}$$

The inner surface of the mold receives a heat flux from the billet, so the condition for matching the temperature distributions in the billet and mold will be

$$\lambda_2(t)\frac{\partial t_2(A, y, \tau)}{\partial x}=q_1, \tag{13}$$

$$\lambda_2(t)\frac{\partial t_2(x, B, \tau)}{\partial y}=q_2, \tag{14}$$

where q_1 and q_2 are defined by (6) and (7), respectively.

It is clear that system (1)–(14) cannot presently be solved analytically, since the condition (3) alone in the two-dimensional case rules out an exact analytical solution. Therefore, the problem was handled numerically.

In order to approximate (3), the continuous function Z(x, y, τ) was replaced by two families of net functions X_j^k and Y_i^k, where X_j^k represents a mobile nodal points corresponding to the point of intersection of the Z(x, y, τ) curve with a coordinate straight line of family j at time kτ, while Y_i^k corresponds to the point of intersection of Z(x, y, τ) with a coordinate straight line of family i. Then the position of the front at any time kτ is determined by the values of the discrete functions X_j^k and X_i^k, which are used in approximating (3)

Then we have the following equations to determine X_j^{k+1} and Y_i^{k+1} on the basis that the temperature near the crystallization front varies only slightly (i.e., any variation in the thermophysical parameters may be neglected):

$$X_j^{k+1}=X_j^k+\frac{T_{cr}-t_{i,j}^k}{X_j^k-ih_x}\cdot\frac{\lambda\tau}{\rho L}\operatorname{tg}(\widehat{n,x}), \tag{15}$$

$$Y_i^{k+1}=Y_i^k+\frac{T_{cr}-t_{i,j}^k}{Y_i^k-jh_y}\cdot\frac{\lambda\tau}{\rho L}\operatorname{tg}(\widehat{n,y}). \tag{16}$$

The nodes within the solidified parts, where (1) applies, are split up into three families Π_1, Π_2, Π_3; the first family covers nodes reasonably remote from the mobile boundary (ones that do not have adjacent mobile nodes). The immobile nodes having adjacent nodes X_j^k, Y_i^k are assigned to the second region, while nodes that transfer to a new phase during a time step are assigned to the third region. The grid function $t_{i,j}^{k+1}$ in the first region is defined by the explicit equation

$$t^{k+1}_{i,j}=t^k_{i,j}+\frac{a_{i,j}\tau}{h_x^2}\left[\left(1+\frac{\lambda^k_{i,j+1}-\lambda^k_{i,j-1}}{4\lambda^k_{i,j}}\right)\left(t^k_{i,j+1}-t^k_{i,j+1}\right)+\left(1-\frac{\lambda^k_{i,j+1}-\lambda^k_{i,j-1}}{4\lambda^k_{i,j}}\right)\left(t^k_{i,j-1}-t^k_{i,j}\right)\right]+\frac{a_{i,j}\tau}{h_y^2}\times$$

$$\times\left[\left(1+\frac{\lambda^k_{i+1,j}-\lambda^k_{i-1,j}}{4\lambda^k_{i,j}}\right)\left(t^k_{i+1,j}-t^k_{i,j}\right)+\left(1-\frac{\lambda^k_{i+1,j}-\lambda^k_{i-1,j}}{4\lambda^k_{i,j}}\right)\left(t^k_{i-1,j}-t^k_{i,j}\right)\right]. \tag{17}$$

In the second region, we have inexplicit equations, for instance, for points adjoining the mobile nodes of the Y_i^k family [6]:

$$t^{k+1}_{i,j}=\left\{t^k_{i,j}\left[\frac{1}{2a\tau}-\frac{1}{h_x^2}\right]+\frac{1}{(j+1)h_y-Y_i^{k+1}}\left[t^{k+1}_{i,j+1}+\frac{T_{cr}}{jh_y-Y_i^{k+1}}\right]+\frac{1}{2h_x^2}\left[t^k_{i+1,j}+t^k_{i-1,j}\right]\right\}:$$

$$:\left\{\frac{1}{2a\tau}+\frac{1}{(j+1)h_y-Y_i^{k+1}}\left[\frac{1}{h_y}+\frac{1}{jh_y-Y_i^{k+1}}\right]\right\}. \tag{18}$$

The temperature at time instant k + 1 for points in the third region may be determined by assuming that the gradient is linear between the boundary nodal point and the crystallization front, i.e., that the relevant equations are

$$t^{k+1}_{i-1}=T_{cr}+\left[\frac{t^{k+1}_{i,j-1}-T_{cr}}{ih_x-X_j^{k+1}}\right]\left[(i-1)h_x-X_j^{k+1}\right], \tag{19}$$

$$t^{k+1}_{i,j-1}=T_{cr}+\left[\frac{t^{k+1}_{i,j}-T_{cr}}{jh_y-Y_i^{k+1}}\right]\left[(j-1)h_y-Y_i^{k+1}\right]. \tag{20}$$

Equations (18)-(20) do not incorporate any change in the thermophysical parameters, since the second and third regions lie reasonably close to the front, where the temperature gradient cannot be large, and therefore the thermophysical parameters must be virtually constant.

The nonlinear boundary conditions of (6) and (7) were realized by iteration on Newton's formula, the final form of the formula for (7) being

$$t^{k,\,s+1}_{n,j}=t^{k,\,s}_{n,j}-\left\{\frac{\lambda}{h_x}\left(t^k_{n,j}-t^{k,\,s}_{n+1,j}\right)+C\left[\frac{\left(t^k_{n,j}-t^{k,\,s}_{n+1,j}\right)}{2}-T\right]+\right.$$

$$\left.+D\left[\frac{\left(t^k_{n,j}-t^{k,\,s}_{n+1,j}\right)}{16}-T\right]\right\}:\left\{\frac{-\lambda}{h_x}-\frac{C}{2}-\frac{D}{4}\left(t^k_{n,j}-t^{k,\,s}_{n+1,j}\right)^3\right\}. \tag{21}$$

Then (6) and (7) for $\tau\leq\tau^*$ correspond to the equations

$$t^{k+1}_{n,j}=\frac{t^k_{n,j}+t^k_{n+1,j}}{1+\dfrac{\lambda^k_{2n,j}}{\lambda^k_{1n,j}}}, \tag{22}$$

$$t_{i,m}^{k+1}=\frac{t_{i,m}^{k}+t_{i,m+1}^{k}}{1+\frac{\lambda_{2m,i}^{k}}{\lambda_{1m,i}^{k}}}, \tag{23}$$

and (5) and (10), which express the adiabatic conditions at the boundaries, correspond to

$$t_{i,-1}^{k}=t_{i,1}^{k} \tag{24}$$

and

$$t_{-1,j}^{k}=t_{1,j}^{k}. \tag{25}$$

The equations of thermal conduction and the boundary conditions for the mold may be simulated in similar fashion.

An ALGOL-60 program was written to solve this system of equations.

A BÉSM-4 computer was used in a series of detailed calculations for 12-ton billets of fully or partly solidified steel. These gave the temperature distributions in the billet and mold, the course of the crystallization front, the time to complete solidification, the necessary heating time, and other information.

Figure 2 shows the temperature at the center and at the surface of the mold for various styles of preparation; the calculations show that there is a prominent minimum in the relationship between the heating time and the temperature (Fig. 3, which has been observed previously in several studies [1, 2, 4]. It is of some practical importance to define the exact set of working parameters corresponding to the minimum. As the heating time is a function of many variables, one has to examine a considerable number of possibilities in order to determine the optimal values, which very much hinders the use of electrical or other analogs for the purpose [4, 8].

Our method employs a fast digital computer, and enabled us to simulate and examine any feasible mode of thermal preparation for rolling, which thus can enable one to define the set of parameters for any detailed working conditions.

Also, the calculations define the conditions under which any particular simplifications are justified, e.g., one designed to provide approximate analytical relationships. For instance, it was found from the calculations that, if the billet is nearly of square cross section, the restriction on the second coordinate has no particular effect on the motion of the front, while the thickness of the solidified portion does not exceed 40% of the size of the billet, which is confirmed by experiment [3]. Subsequently, the solidification accelerates substantially, and it may be described in terms of a one-dimensional cylindrical model subject to certain assumptions. Similarly, one can estimate the time needed to reach the regular cooling stage in the billet or mold.

These characteristic features provide reasonably sound means of selecting approximate analytical relationships for the major stages.

The calculations by our method agree with other simulation results and with measurements [8] on the temperatures during crystallization of actual billets.

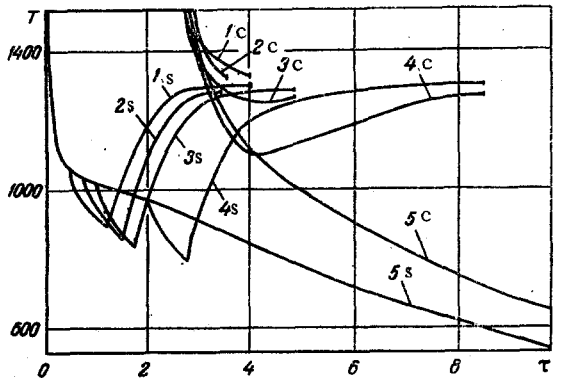

Fig. 2. Temperatures at surface and center as functions of time in various modes of preparation: 1) τ_c = 30 min, τ_s = 45 min; 2) 45 and 90 min, respectively; 3) 60 and 105 min; 4) 120 and 165 min; 5) cooling in mold. T in °C, τ in h.

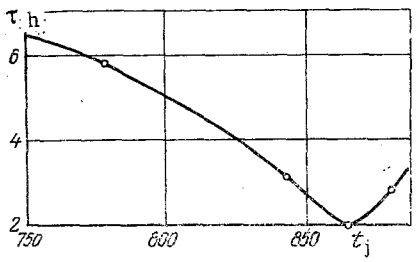

Fig. 3. Heating time τ_h as a function of jacket temperature τ_j.

NOTATION

t, temperature; λ, thermal conductivity; c, specific heat; ρ, density; 1, ingot; 2, mold; x, y, Cartesian coordinates; τ, time; time step; q, heat flux density; L, latent heat of crystallization; t_s, t_a, t_b, temperature of internal surface of mold, air, and bath, respectively; τ^*, τ_c, τ_{in}, τ_r, times of breakaway, casting removal, start of heating, and readiness for rolling; σ_1, α_1, emissivity in visible region and heat-transfer coefficient for air cooling; σ_2, α_2, the same for bath heating; σ_3, α_3, the same for the cooling in the mold; i, j, numbers of nodes in a region; k, time step; n, m, numbers of nodes x and y, respectively.

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